Technical Milestone Report

Robotic Unicycle: Mechanics & Control

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January 2005

1 Abstract

A robotic unicycle has been partially built and is being developed jointly by Mark Mellors and Andrew Lamb. The project applies and develops knowledge learnt at CUED. The dynamics of a unicycle has been analysed and modelled using simulink. The Pro/E model was developed using lessons learnt from the dynamics modelling. The unicycle will be used to encourage others into engineering

2 Introduction

Balancing a unicycle is difficult for a human rider; we are producing one that does it by itself. Because of the scope and scale of the work required the project is a collaboration of Andrew Lamb (electronics and control) and Mark Mellors (mechanics and control). A unicycle is a unique form of transport that requires a great deal of skill and balance to ride. Because it is so unstable it can be very manoeuvrable and is entertaining to watch and ride. The unicycle will be a showcase of what can be achieved through the application of engineering.

3 Project Aims

- Investigate the mechanisms involved in the balancing of unicycles.
- To apply skills learnt in control, tribology, dynamics and structural lectures to this problem.
- To develop an understanding of control systems.
- To become familiar with the hardware of control systems and their interfaces.
- To have an impressive project product that can lead to future research.

4 Context

The dynamics of a rolling coin and an inverted pendulum are analysed in lectures. This project is an extension of both of these. The author has an interest in robotics as well as riding unicycles. The project will be a combination of theory and practical knowledge. Previous attempts by others at building robotic unicycles have not been particularly successful.

5 Plan

- Assemble equations of motion.
- Propose a method of control.
- Develop a design and specification.
- Build a unicycle.
- Model the real unicycle.
- Converge model with unicycle dynamics.
- Use model to develop real mechanical control system and vice versa.
- Demonstrate the real working robotic unicycle at robotics competitions.

This plan overview has been broken down into more detail and presented in Gantt chart form. Due to the nature of the project the mechanical, control and electrical plans are interdependent. The Gantt chart presented in Lamb's report considers this and represents the joint plan for both projects.

6 Progress

- Development of principal equations of motion

The motion of the unicycle was analysed using d'Alembert's approach. Gyroscopic effects and the inertia of the wheel and drive train were assumed negligible at this stage. All external influences and irregularities (such as friction) have been ignored.

Equations for pitch
By resolving moments

$$mgl\sin q - T = I_q \ddot{q}$$

By resolving horizontally

$$m\left(\ddot{x} + l\ddot{q}\cos q - l\dot{q}^2 \sin q\right) + f = 0$$

Equations for yaw
By resolving moments

$$I_j \dot{j} = Q$$

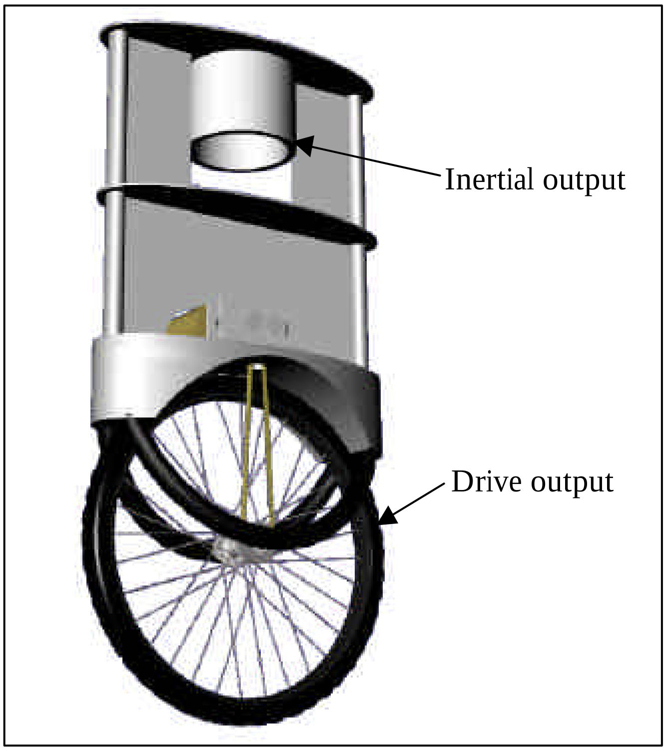
Equations for roll
By resolving moments

$$I_f \ddot{f} = mhv\dot{j} + mgl\sin f$$

All definitions are covered in section 10

These equations have been verified through simple experiments and comparison with others results. Schoonwinkel takes several different approaches to deriving these equations, but ultimately they all take the same form as those shown above.

- Principal outputs for control



The outputs were chosen so that they represented the human method of control. The inertial output represents a rider's arms and trunk, which are rotated in order to twist the unicycle. Balance in pitch will be achieved by driving the wheel back under the unicycle using the drive output. Balance in roll will be achieved through two methods. The first makes use of centripetal force. If the unicycle begins to lean over too far whilst travelling in a circle, the inertial output will be used to force the unicycle into a tighter turn. This will increase the centripetal force on the unicycle, which will pull it into a more upright position and allow the angle of roll to be maintained. However, when moving slowly this will only provide limited control so a second control method is needed. This involves turning the unicycle quickly into the direction of lean using the inertial output, then driving the wheel back under the unicycle and forcing it into an upright position. The inertia output turns the unicycle by utilising the Newton's third law: if the inertia is spun clockwise, the unicycle will be forced to spin anti-clockwise.

- Control system proposed

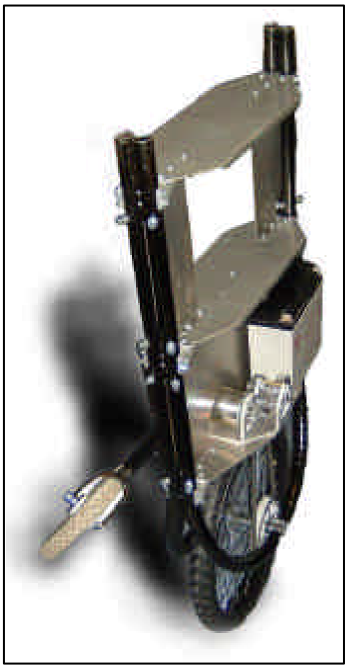
The control system diagram is presented in the appendix. The general method of control is that a velocity is demanded and a suitable lean angle is calculated. This lean is achieved by using the outputs to apply a suitable torque. This method of control is being evaluated with the Simulink model.

- Pro/Engineer model constructed.

The unicycle was designed in detail in Pro/Engineer after making some preliminary calculations. The equations of motion suggested that a high centre of gravity would make the time constants larger, thus making balancing easier. They also showed that the inertia about the vertical axis would need to be minimised in order to limit the power required to turn. From the aims of the project, it was decided that the unicycle should have similar dimensions to a human ridden unicycle. The power requirements were estimated using a basic analysis and from this the fundamental components were sized. Using this data a tall frame was designed that was only sufficiently wide to accommodate the components. Internal panels were provided for mounting the main components. The gearboxes for the drive and inertial outputs were sized to provide the correct gear ratio and to suffer minimal wear over the life of the unicycle. A belt drive was used for the final ratio, as this was the solution with the smallest backlash. Any backlash will cause control difficulties.

At this point, the moment of inertia (MOI) of the unicycle was estimated (the final result is given in the appendix). In the process of reducing the inertia, the gearboxes and battery were moved as close to the centre of the unicycle as possible. The main component of the inertia about the vertical axis was now the outer panels, originally 3mm aluminium weighing nearly 6kg. By changing to CFRP the weight was reduced to 1.2kg and the overall inertia halved while sacrificing little in strength.

- Unicycle Built.



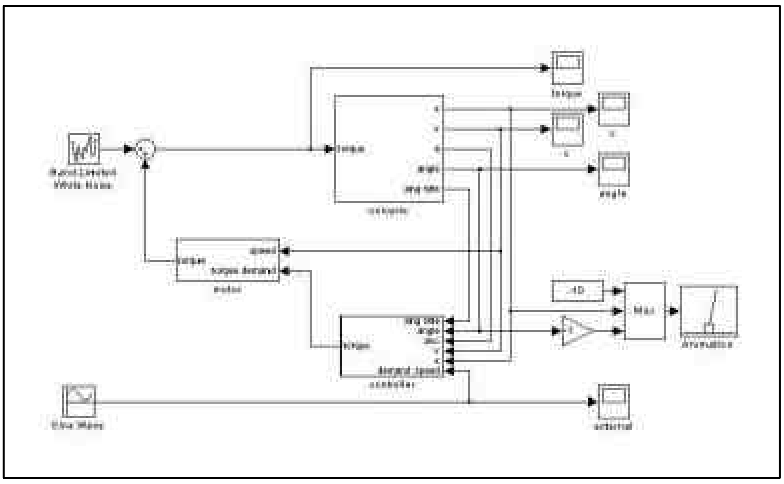
The unicycle is mechanically ready to be balanced in pitch. It still requires mounts for the electronics and sensors. The smaller white supporting wheel is temporarily mounted in order to provide roll stability while the pitch controller is developed. The drive system provides sufficient torque to wheel spin and the top speed is a fast walking pace. The construction was relatively straightforward with little difficulties. This was mostly due to the detailed design and the planning that had taken place. The inertial output has been designed and construction will start shortly. This will involve building a similar gearbox to the drive one and mounting it to the top level of the unicycle. The inertia itself will be a large, thick walled steel pipe, which will rotate around the gearbox.

Initially the unicycle will be tested with the supporting wheel and suspended from a galley. The unicycle will be free to move but will not be able to hit the floor. This my require a harness to be constructed.

- Recorded motion of unicycle and rider.

A rate gyroscope was attached to a human ridden unicycle while the rider was idling on the spot. This confirmed the typical accelerations and speeds of a human ridden unicycle. When the robotic unicycle is approaching roll stability the human ridden unicycle will be monitored again to compare their differing approaches to stability and control.

- Simulink model developed.



The equations of motion of the unicycle have been used to develop a model within Simulink. A model of the control system detailed in the appendix has also been constructed then developed within Simulink. The pitch controller is now very robust and provides fast reactions. The roll controller currently provides marginal stability, but is still being developed.

When the sensors are mounted on the robotic unicycle the behaviour will be monitored on all tests, whether the unicycle balances or not. This data will then be used to confirm and refine the unicycle model within Simulink.

7 Conclusion

The project is progressing well and is on track to be balancing by the end of term. Even the stationary unicycle is impressive and is generating interest in a wide range of people. Work needs to continue at the same persistent rate for the project to be successful. Latest progress can be followed at www.roboticunicycle.info.

8 Acknowledgments

Jan Maciejowski's expertise continues to be been a vital aid for the project in his role as supervisor.
The unicycle was built by Mellors and Lamb in the engineering departments workshop from technical drawings constructed from the pro/engineer model. Richard Christmas, Alistair Ross and Peter Long provided guidance. The CFRP panels were constructed by Mellors at Carbon Concepts. Dr Nick Mcleod and Dr John Dominic and thanked for there assistance and generosity.

9 References

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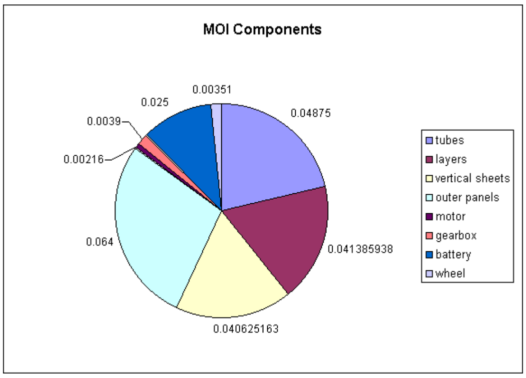
Zaiquan Sheng and Kazuo Yamafuji, "Postural Stability of a Human Riding a Unicycle and Its Emulation by a Robot," IEEE transactions on robotics and automation, VOL 13, No. 5, October 1997]
R. C. Johnson, "Unicycles and Bifurcations," October 2002

10 Definitions

f yaw angle about a vertical axis
? pitch angle about the wheel axis
f roll angle about the front-back axis
v wheel velocity
m mass of unicycle
g acceleration due to gravity
l height of centre of mass of the unicycle

T Wheel Toque
Q inertial output torque
$I_?$ unicycle inertia about the pitch axis
I_f unicycle inertia about the yaw axis
I_f unicycle inertia about the roll axis
x distance travelled forwards
f tyre friction force

11 Appendix



Control Diagram